Wave Motion and Wave Actions: Part I  Wave Motion

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3 NON-LINEAR EFFECTS

3.1 Higher order wave theories
Airy (first-order Stokes) theory: free surface boundary conditions were applied at still water level $z = 0$, and the nonlinear terms omitted. Higher order theories may retain these terms approximately. They use the first order solution as an estimate of the free surface and in the non-linear terms for the second order and so on. These solutions should give more accurate wave shape, $\eta(x, t)$, and velocity but this is not always the case. The solution is in the form of sine and cosine series with very messy equations for the coefficients (the usual form is “simplified” by using terms which are products of sines and cosines)

3.2 Stream function theories
In place of the potential function, $\phi$, the stream function, $\psi$, may be used. By definition

$$\psi = \frac{\partial \phi}{\partial z} \quad \text{and} \quad w = -\frac{\partial \psi}{\partial x}$$

For waves of constant form, the flow pattern may be made steady (unchanging with time) by moving the frame of reference at the speed of the waves, $c$. The sea bed and the water surface are then streamlines. This means that the kinematic boundary condition at the sea surface is exactly satisfied and does lead to improved accuracy for a given order of solution.

3.3 Cnoidal wave theories
Finite amplitude waves of permanent form in shallow water, but not close to breaking, are well described by cnoidal wave theory. The theory is named for the elliptic functions which are solutions of the Kortweg-de Vries equation. At one limit the solutions reduce to first-order Stokes and at the other limit to solitary wave theory. The solutions require mathematical tables for evaluation but are reproduced in Wiegel (1964) and the Shore Protection Manual. Related to the cnoidal solution, but more general in its scope is the voceoidal wave theory of the National Research Institute for Oceanography in Stellenbosch (Swart and Loubser, 1979).

3.4 Higher order Stokes wave theories
Dean (1974) presented tabulated values of the coefficients of higher order stream function solutions for 40 sample cases. The solutions are given in the form:

$$w(\theta, s) = -\sum_{n=1}^{N} X(n) \left( \frac{2 \pi s}{\lambda} \right) \cosh \left( \frac{2 \pi s}{\lambda} \right) \cos n \theta$$

where $\theta$ is the phase angle of the wave, corresponding to the quantity $\left( k \theta - \sigma t \right)$ and $s = d + z$.

By interpolating on Dean’s charts for the $X(n)$ values as functions of $h/\lambda o$ and $d/\lambda o$ (where $\lambda o$ is the deepwater wavelength) the velocity and other variables may be determined. Chaplin modified Dean’s approach to improve accuracy for steep waves in shallow water (Chaplin and Anastasios (1980), Chaplin (1980)).
3 NON-LINEAR EFFECTS

3.4 Fenton wave theory

Stokes, stream function and cnoidal theories simplify the equations and boundary conditions, then obtain exact solutions to these approximate equations. Fenton wave theory finds approximate solutions to the exact equations and boundary conditions.

In Fenton theory the user specifies the number of points in one wave length at which the equations are to be satisfied. At all other points the solution is less exact but still accurate.

With Fenton theory the user defines the number of sine and cosine terms used (typically 5 to 7), and hence the accuracy. It is easy to increase this number to see if it affects the answer significantly.

Fenton theory is more accurate with fewer terms than any of the other higher order theories, and the functions are simpler.

3.5 Regions of validity of wave theories

In practice the second-order Stokes theory is sometimes less accurate than the first order. In design work requiring high accuracy fifth-order Stokes theory has often been used.

In stream function theory the kinematic boundary condition at the sea surface is exactly satisfied and does lead to improved accuracy for a given order of solution.

Fenton theory yields a more accurate result with fewer terms than any of the other higher order theories.

The most accurate procedure is the boundary or surface integral method of Longuet-Higgins and Cokelet (1976) but requires numerical solution for each case.

3.6 Wave breaking

Broadly speaking, waves break because they become too energetic for their speed of propagation.

In deep water, an individual wave may gain energy from work done by the wind on the water surface and by interactions between waves of different frequencies and directions.

In deep water, Michell in 1895 deduced that for a wave to break

\[ \frac{H}{gT} \geq 0.142 \]

The theoretical wave crest at breaking is a cusp of angle 120 degrees.

There has been a lot of work but little progress in predicting deep water wave breaking and numerical wave propagation models use empirical rules.

In shoaling water the wave energy is concentrated by the reduction in depth and the consequent reduction in wave length.

Munk, from solitary wave theory, obtained

\[ \text{breaker index} = \frac{H_b}{d_b} = 0.78 \]

Which is adequate as a rough guide only; lower values apply generally.

More exactly, the wave steepness at breaking depends upon the relative water depth and the bed slope, \( m \).

Large scale flume tests provided data for figures given in the Shore Protection Manual (CERC 1984) and the ACES software.
3 NON-LINEAR EFFECTS

3.6 Wave breaking

Breaker Types:

Plunging breaker – wave steep, beach steep

Spilling breaker – wave and beach slopes moderate

Surging or collapsing breaker – beach slope very steep
3.6 Wave breaking

Breaker Types: Surging or collapsing breaker – beach slope very steep.